PHY 114 PROBLEM SET 6

26.44. a) The time constant ($\tau$) of a circuit is given as-

$$\tau = RC$$

$$C = \frac{\tau}{R} = \frac{24\times10^{-6}\,s}{15\times10^{3}\,\Omega} = 1.6\times10^{-9}\,F$$

b) The total EMF of the cell is 24 V and voltage across the resistor is 16 V.

Hence voltage across capacitor is $(24-16) = 8\,V$.

The eq. showing the charging process of a capacitor is-

$$V = E(1-e^{-t/\tau})$$

$$\frac{V}{E} = 1-e^{-t/\tau}$$

$$e^{-t/\tau} = 1-\frac{V}{E} = 1-\frac{8}{24} = 1-\frac{1}{3} = \frac{2}{3}$$

$$\therefore -\frac{t}{\tau} = \ln\left(\frac{2}{3}\right) \qquad \therefore t = -\tau\ln\left(\frac{2}{3}\right) = -24\times10^{-6}\ln\left(\frac{2}{3}\right) = 9.73\times10^{-6}\,s$$

26.46. Energy stored in a capacitor $= \frac{1}{2}\frac{Q^2}{C}$

The charge of the capacitor is $Q(t) = CE(1-e^{-t/\tau})$

$$U_i = \frac{1}{2}\frac{Q^2(0)}{C}, \quad U_f = \frac{1}{2}\frac{Q^2(t)}{C} = 0.75\,U_i$$

$$\therefore \frac{U_f}{U_i} = \frac{Q^2(t)}{Q^2(0)} = 0.75$$

$$\frac{C^2E^2(1-e^{-t/\tau})^2}{C^2E^2(1-0)^2} = (1-e^{-t/\tau})^2 = 0.75$$

$$1-e^{-t/\tau} = \sqrt{0.75} \qquad e^{-t/\tau} = (1-\sqrt{0.75})$$

$$-\frac{t}{\tau} = \ln(1-\sqrt{0.75}) \qquad \therefore t = -\tau\ln(1-\sqrt{0.75}) = 2.01\tau$$

26.48. The voltage of a capacitor undergoing a discharging is given as -

$$V_c = V_0 e^{-t/RC}$$

$$V_c = \frac{0.10}{100}V_0$$

$$\frac{0.10}{100}V_0 = V_0 e^{-t/RC} \qquad e^{-t/RC} = 0.0010$$

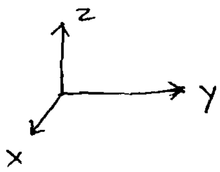
$$-\frac{t}{RC} = \ln(0.0010) \qquad \therefore t = -(8.7\times10^{3}\,\Omega)(3\times10^{-6}\,F)\ln(0.0010)$$

$$= 0.18\,s$$

27.2. The force on a wire placed in a magnetic field is –

$\vec{F} = I\vec{l}\times\vec{B} = I\,l\,B\sin\theta = (150\text{ A})(240\text{ m})(5\times10^{-5}\text{ T})\sin 68^\circ = 1.7\text{ N}$

27.16. Let us choose our coordinate system as below –




$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

a) $\vec{V} = V\,\hat{i}$, $\vec{B} = -B\,\hat{k}$

$\vec{F} = (-q)\,\vec{V}\times\vec{B} = qVB\,(\hat{i}\times\hat{k}) = -qVB\,\hat{j}$ (left)

b) $\vec{V} = -V\hat{k}$, $\vec{B} = -B\hat{i}$

$\vec{F} = (-q)\,\vec{V}\times\vec{B} = -qVB\,(\hat{k}\times\hat{i}) = -qVB\,\hat{j}$ (left)

c) $\vec{V} = -V\hat{i}$, $\vec{B} = B\hat{j}$

$\vec{F} = (-q)\,(\vec{V}\times\vec{B}) = qVB\,(\hat{i}\times\hat{j}) = qVB\,\hat{k}$ (upward)

d) $\vec{V} = V\hat{j}$, $\vec{B} = B\,\hat{k}$

$\vec{F} = (-q)\,\vec{V}\times\vec{B} = -qVB\,(\hat{j}\times\hat{k}) = -qVB\,\hat{i}$ (inward into paper)

e) $\vec{V} = -V\hat{j}$, $\vec{B} = +B\hat{j}$

$\vec{F} = (-q)\,\vec{V}\times\vec{B} = qVB\,(\hat{j}\times\hat{j}) = 0$

f) $\vec{V} = -V\hat{j}$, $\vec{B} = B\hat{i}$

$\vec{F} = (-q)\,\vec{V}\times\vec{B} = qVB\,(\hat{j}\times\hat{i}) = -qVB\,\hat{k}$ (downward)

27.18. Since the electron goes undeflected, the electric force must compensate for the magnetic force acting on the ~~partic~~ electron.

$F_e = F_B$

$qE = qVB$

$\therefore V = \dfrac{E}{B} = \dfrac{8.8\times10^{3}\text{ V/m}}{7.5\times10^{-3}\text{ T}} = 1.173\times10^{6}\text{ m/s}$

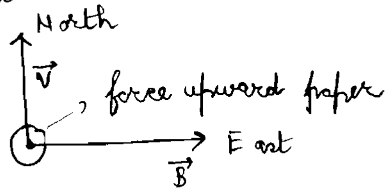
If the electric field is turned off, the magnetic force will deflect the particle and make it move in a circle. The centripetal force will balance magnetic force now.

$F_B = qVB = \dfrac{mV^2}{r}$

$r = \dfrac{mV}{qB} = \dfrac{(9.11\times10^{-31}\text{ Kg})(1.173\times10^{6}\text{ m/s})}{(1.6\times10^{-19}\text{ C})(7.5\times10^{-3}\text{ T})} = 8.9\times10^{-4}\text{ m}$

27.24. $F_B = qVB$ $\quad \therefore B = \dfrac{F_B}{qV} = \dfrac{8.2\times10^{-13}N}{(1.6\times10^{-19}c)(2.8\times10^{6}\,m/s)} = 1.8\,T$

The direction of the magnetic field must be along east applying the right hand rule.




27.28. The perpendicular component of velocity to magnetic field contributes to magnetic force.

$$F = qV_{\perp}B\sin\theta = m\frac{V_{\perp}^2}{r}$$

$$r = \frac{mV_{\perp}}{qB} = \frac{(9.11\times10^{-31}\,Kg)\,(3\times10^{6}\,m/s)\,\sin 45^\circ}{(1.6\times10^{-19}c)\,(0.28\,T)} = 4.314\times10^{-5}\,m$$

The parallel component of velocity remains unchanged and pitch will be the distance travelled due to that velocity in one cycle of time period T.

$$T = \frac{2\pi r}{V_{\perp}} = 2\pi\,\frac{mV_{\perp}}{qB}\cdot\frac{1}{V_{\perp}} = \frac{2\pi m}{qB}$$

$$p = V_{\parallel}T = V\cos 45^\circ\left(\frac{2\pi m}{qB}\right) = \frac{(3\times10^{6}\,m/s)\cos 45^\circ\times 2\pi\,(9.11\times10^{-31}Kg)}{(1.6\times10^{-19}c)(0.28T)}$$

$$= 2.7\times10^{-4}\,m$$

27.51. The magnetic force causes ions to move in a circle.

$$qVB = \frac{mV^2}{r} \qquad m = \frac{qBr}{V}$$

$$\frac{m}{r} = \frac{qB}{V} = \text{const.}$$ ($\because$ all particles are of same charge and in same magnetic field)

$$= \frac{76\,U}{22.8\,cm}$$

$$\therefore m_{21} = 21\times\frac{76U}{22.8} = 70U \qquad m_{21.6} = 21.6\times\frac{76U}{22.8} = 72U$$

$$m_{21.9} = 21.9\times\frac{76U}{22.8} = 73U \qquad m_{22.2} = 22.2\times\frac{76U}{22.8} = 74U$$

27.66. a) The frequency of the voltage has to match the frequency of revolution of the particle to have a synchronised motion.

Time period of motion $= \dfrac{2\pi r}{V} = \dfrac{\text{Dist. or Circumference of circle}}{\text{Velocity}}$

$= T$

For centripetal acceleration -

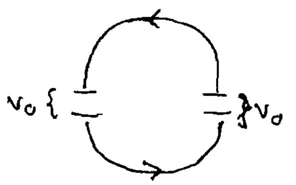
$$\frac{mv^2}{r} = qVB \qquad \therefore r = \frac{mV}{qB}$$

$$f = \frac{1}{T} = \frac{V}{2\pi r} = \frac{V}{2\pi} \times \frac{qB}{mV} = \frac{qB}{2\pi m}$$

b) The small gap has an electric field with increases the kinetic energy of the particle.

$$F = qE = q\frac{V_0}{d}$$

$$K.E. = F.d = qV_0$$




In one circular motion the particle passes through the gaps twice. Hence net increase in energy is $2qV_0$

c) $$K_{max} = \frac{1}{2} m V_{max}^2 = \frac{1}{2} m \left(\frac{r_{max} qB}{m}\right)^2 = \frac{1}{2} \frac{r_{max}^2 q^2 B^2}{m}$$

$$= \frac{1}{2} \frac{(0.5\,m)^2 (1.6 \times 10^{-19} C)^2 (0.6\,T)^2}{1.67 \times 10^{-27} Kg}$$

$$= 6.898 \times 10^{-13} J \frac{(1 eV)}{1.6 \times 10^{-19} J} \times \frac{1 MeV}{10^6 eV} = 4.3 MeV$$

The max. energy is gained in the outermost rim ($r_{max}$) of cyclotron.